BEAM: A New Simulation to Evaluate Strategies and Forces for Military Campaigns

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ABSTRACT

The Bilateral Enterprise Analysis Model (BEAM) is a new joint campaign tool that enables easy analysis of military theater strategy, force structure, and/or infrastructure changes. BEAM has an enterprise resolution, which is less detailed than campaign simulations. Within BEAM, missions are the main modeling entities. BEAM employs a novel approach to modeling uncertainty in that it models the statistical distributions of all assets within regions. Applying a Quasi-Monte Carlo approach, BEAM conducts a design of experiments and initiates a simulation thread at each design point for each simulated day. Within each thread, each opponent's sequence of algorithms:

1) *Accounts for their perceptions of both sides;*
2) *Updates their prediction of their adversary's military strategy;*
3) *Conducts fictitious play with each side adapting against their perception of their adversary's missions;*
4) *Adjudicates the opponents' selected missions, accounting for uncertainty of mission results for offensive, defensive, and targeted assets; and*
5) *Conducts battle damage assessment to determine their perceptions of the current situation.*

Probability and outcome states associated with the assessing side having achieved all their phase ends (military objectives) are stored until the following phase commences. The repeated application of the simulation threads produces statistical distribution of outcomes with a single pass through the simulated time (with no need for replications). The United States (US) has employed BEAM in a large trade-space study and is using it in three additional studies. BEAM has a wargaming mode where players may modify their military strategies. The US is distributing BEAM for free to US organizations and NATO nations. NATO is intending to include BEAM in the NATO Next Generation Modelling and Simulation (M&S) Architecture. In this article, we provide an in-depth description of BEAM's algorithms and provide some examples of how analysts may use to evaluate military force and strategy in a theater war.

Keywords: Campaign analysis; Force structure; Military effectiveness; Military strategy; Quasi-Monte Carlo; Thread simulation

1.0 INTRODUCTION

Tecott and Halterman [1] make a compelling case for the importance of military campaign analysis. Issues of military strategy, force mixes, and infrastructure should be investigated. The US Joint Staff defines military strategy for a combatant command's theater goals specified through ends (desired outcomes), ways (how the ends will be achieved), means (the resources employed), and risks (avoiding undesired results) [2]. Forces constitute the combat and supporting units. Infrastructure includes the nations allowing their ports and airfields to support operations. Our focus in this article is analytically evaluating military theater campaigns.

Many combat models are available to the defense analytic community to assess military campaigns and engagements; however, a gap exists with respect to the ability to analyze military campaigns in a timely manner in support of senior leader decision making. To address this gap, we explored approaches with less resolution that integrated more breadth (combat domains) and more scope (functionalities) than current campaign models. The Bilateral Enterprise Analysis Model (BEAM), which enables quantitative analysis of military strategy, forces, and infrastructure, is the result of our exploration. We employ a new analytic approach in BEAM that enables the development of the distribution of combat outcomes with one pass through simulated time. As a result, BEAM provides defense analysts with the ability to quickly search the extensive trade-space of military strategies, force mixes, and available infrastructure in major theater conflicts.

US military organizations, US defense contractors, and the North Atlantic Treaty Organization (NATO) are evaluating BEAM. US and NATO leaders require the ability to evaluate alternative military strategies for theater-level campaigns; until now, military planners have relied predominantly on wargames rather than quantitative analysis. BEAM is not intended to replace strategic wargames, however, using BEAM for pre-, post-, and in-game analysis and support may enhance both the wargame itself as well as the results and insights gained from the wargame. Additionally, BEAM has the unique ability to evaluate the extensive strategy and force mix trade-space, which can provide insights beyond what wargames alone can produce. BEAM should be used in conjunction with campaign models, which can verify insights derived from analysis with BEAM.

2.0 BEAM RELATIONSHIP TO OTHER COMBAT MODELS

DeGrange and Darrow [3] describe the analytic process for conducting a study. Brown [4] in the *Analytics Body of Knowledge* provides an overview of modeling techniques applied in analysis. Law [5] present a taxonomy of models, in which, combat simulations are almost exclusively classified as dynamic, stochastic models [6]. Hodson [7] categorizes the various military applications of simulation, where we focus on simulations that represent the entire environment. Hill and Tolk [8] summarize the history of applying simulation to military situations. The military has led the advancement of simulations [9]. Hill, Miller, and McIntyre [10] were the first to publish a hierarchy of combat models based on their resolution. The update to their hierarchy added two levels of resolution and breadth beyond campaign models [11]. Lu et al. [12] provide a recent evaluation of military simulations. Currently, the Synthetic Theater Operations Research Model (STORM) [13], [14], [15] and Joint Integrated Contingency Model (JICM) [16], [17] are the predominate theater campaign simulations. BEAM, which has less resolution, is an enterprise simulation that should be used in conjunction with campaign models. Sweetser and Bexfield [18] summarize the current challenges of incorporating cyber, artificial intelligence, and command and control in current campaign analysis.

BEAM is a new joint simulation of military theater campaigns (operational level of war) [19], [20], [21]. It varies from traditional campaign models in that it is a low-resolution campaign modeling tool that can rapidly evaluate military campaign scenarios, including military strategies against an adaptive adversary. Traditional Department of Defense (DoD) campaign modeling tools have a much higher resolution, so sensitivity analyses and changes to the models can take months to complete. BEAM allows an analyst to adjust the scenario parameters quickly and provide insights into different campaign strategies. BEAM models both friendly and adversary adapting to best achieve their military strategies. BEAM rapidly assesses the campaign impacts of scenario changes, such as force structure and infrastructure including base and port access. BEAM models mission allocations and theater outcomes through an aggregated representation of geography, systems, resources, and missions. The model may be used as a filter for follow-on, higher resolution exploration using traditional campaign models to refine the estimated theater outcomes.

2.1 BEAM Data Structure

Before describing how campaigns are modeled and the detailed algorithms used in BEAM, one should understand four of the major building blocks in the BEAM data structure: assets, sub-packages, packages, and missions. While the structure and data for these building blocks are already present in BEAM, analysts must understand their interactions to build a successful representative scenario.

- Assets are the smallest building block and include anything that affects outcomes (units, combat systems, munitions, fuel, land, roads, runways, ports, and other supporting systems or supplies). Additionally, all assets are potential targets for the adversary.
- Sub-packages are made up of the smallest possible combination of assets. For example, an air refueling sub-package requires three assets: a tanker, air fuel, and a runway. All assets from a sub-package must originate from the same region. So, if a tanker and a runway exist in a region with no air fuel, the assets will be targetable by the enemy, but not useable for missions until air fuel becomes available in that region. Sub-packages ensure assets are allocated in commensurate quantities.
- Packages are made up of sub-packages. Sub-packages can come from different regions. The complete list of packages in BEAM should include all combinations of sub-packages that can be paired together to conduct a mission. For example, a bombing package would have a sub-package that requires appropriate proportions of a bomber aircraft unit, runway, fuel, munitions, and satellite communications. An additional sub-package, possibly from another region, supplies the tanker support and itself requires portions of a tanker unit, runway, and fuel. Secondary sub-packages, such as air escort or cyber combat, are not required; however, including optional sub-packages increases the effectiveness or survivability of assets in the mission. Allocations in BEAM are made at the package level.
- Missions are the primary modeling entity in BEAM; each mission contains one offensive package, an opposing defensive package, and a target package.

BEAM includes a mission effectiveness database. Modelers may add anything that affects combat units or systems in this construct (even non-kinetic effects like electronic warfare and cyber-attacks) and incorporate it into BEAM. The stored mission outcomes are the expected losses along with their variability for the attacking, defending, and targeted assets. The current database has nearly a million missions that vary the type and quantity of assets. Since BEAM only has mission effectiveness data and does not have any system parameters or system performance data, BEAM runs and outputs may avoid some limitations of sensitive and highly classified system data [21].

2.2 Modeling a Scenario in BEAM

Within BEAM, a military strategy is represented as prioritized objectives by combat phase. The combined forces modeled within BEAM include ground, maritime, air, and space units including systems from various countries, aligned in two opposing sides. BEAM does not use specific coordinates, but rather calculates distances from the midpoint of user-defined geographic regions, which are built in layers on an interactive map. Additionally, an extraterrestrial region represents the unique aspects of space systems. Once regions are built, the modeler defines the regional force structures by adding assets, asset quantities, their arrival time, and asset Intelligence, Surveillance, and Reconnaissance (ISR) properties of their detection vulnerability.

The modeling focus on multi-domain missions enables BEAM to easily vary and search military strategies. Within BEAM, military strategies are a collection of military objectives defined by ends, ways, means, and risks specified for each phase of combat. Each end specifies either a condition, such as air superiority, or an asset or asset group to control or degrade to a specified threshold. Users specify the ends by region and assign a priority weight relative to other ends in the same combat phase. BEAM's allocation algorithm optimizes achievement of these weighted ends. Analysts may further refine the strategy by linking ways and means to the ends. A way specifies a particular mission group (such as Air Strike, Missile Strike, or Ground Combat) that must be used to achieve the ends. Similarly, a means dictates the specific asset that must be used (or cannot be used) to achieve a particular end. The final strategy element that the analyst may employ are risks. Risks allow the user to limit the loss of certain assets within a combat phase. Figure 1 shows BEAM inputs for a Phase 1 Blue strategy that seeks to obtain air supremacy over the region "Red Land" ("Air Strike" mission ends) via Air Strike ("Air Attack" ways), excluding the Armor asset ("No Armor" means), and protecting the F-35 ("F-35" risk). The analyst uses this construct to model the Red side, the Blue side, and the perceptions for each side (such as Red's perception of Blue's strategy and vice versa). BEAM uses perceptions to allocate resources to missions, so more accurate perceptions lead to more effective resource allocations.

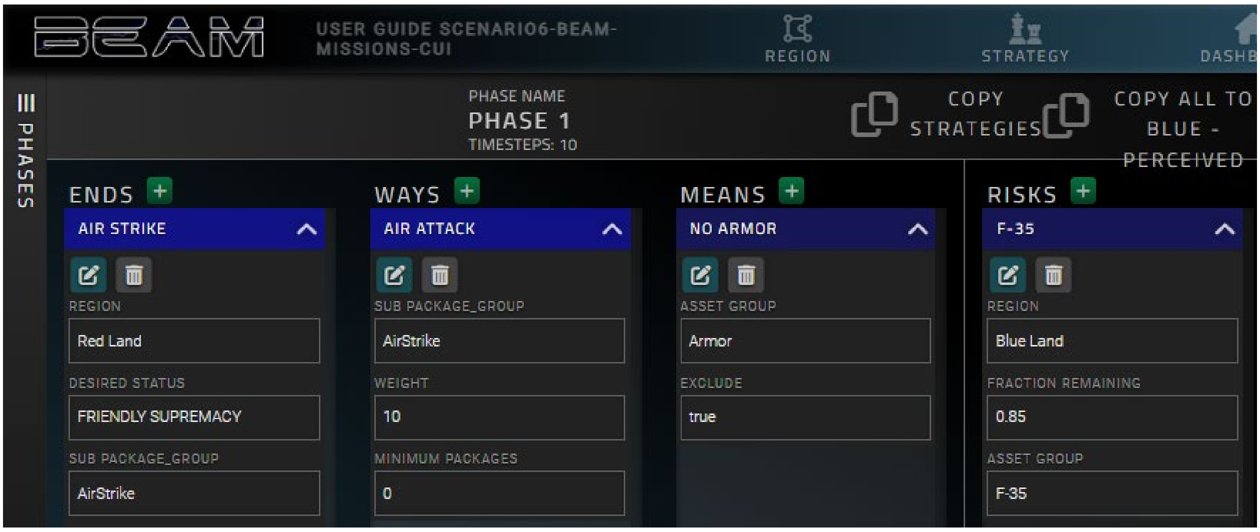


Figure 1: Phase 1 strategy with ends, ways, means, and risk.

2.3 BEAM Analytics

Once the scenario is complete, with force structure and strategy defined, the analyst can run the model. The analyst has the option of performing a single model excursion or conducting a Design of Experiments (DoE). BEAM's built-in parametric analysis capability creates a full factorial design; alternatively, analysts may create a run matrix in Microsoft Excel and import it into BEAM for a more tailored design.

2.3.1 Wargame Mode

BEAM's approach of modeling military strategy, including both sides' ability to adapt, enables us to quickly evaluate different scenarios and perform sensitivity analyses, increasing our responsiveness to senior leader questions and concerns. Currently, wargamers address many of these issues to the best of their ability, however, BEAM may be able to assist in wargame planning and execution using the wargame mode in BEAM. This feature allows the user to *play* a turn based on a user-defined time-step, export the results as a scenario, adjust the military strategy, and play the next turn. This can be repeated as many times as necessary to simulate or support a wargame.

2.3.2 BEAM Output Data

BEAM outputs considerable data for analysts to investigate. Output includes the expected quantities of forces by region along with the quantities of missions selected within each thread for each simulated day. A Microsoft Excel-based BEAM dashboard, shown in Figure 2, allows users to import BEAM output files to view prepared graphs of the consolidated assets, targets, and missions. Alternatively, users may analyze the various output files independently.

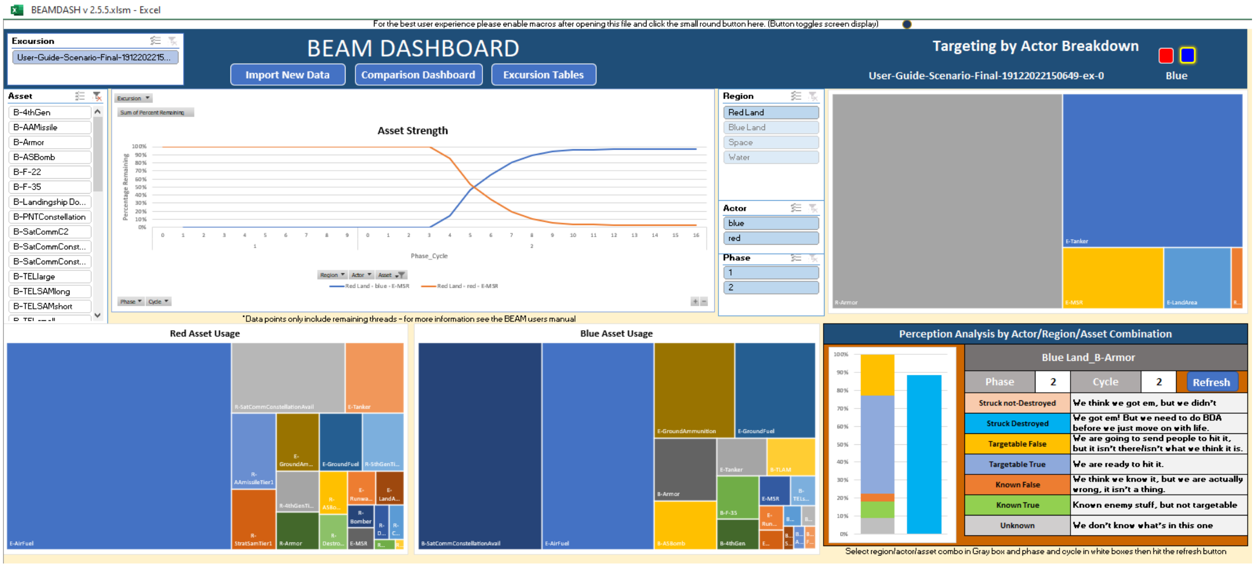


Figure 2: BEAM dashboard main screen.

3.0 THE NEW APPROACHES IN BEAM

BEAM is unique compared to current models in two significant ways: its resolution and its analytic technique. BEAM is an enterprise-level model in that it is considerably more aggregate in resolution than campaign analysis [11]. BEAM models the statistical distributions of assets in geographic regions, does not model individual systems or specific locations, and does not track the trajectories that lead to being at a particular state.

3.1 BEAM Resolution

Campaign models, including STORM and JICM, main modeling entities are combat units and platforms. In contrast, BEAM models the statistical distributions of assets within large geographic regions. Furthermore, BEAM's primary modeling entities are missions which consist of offensive, defensive, and targeted assets. Since missions causes effects that relate to achieving the military strategy ends, BEAM is well suited for evaluating various phase objectives within a combat theater of operations.

BEAM applies the Markov assumption [22] since it only tracks the statistical distributions for the time being simulated and does not retain the trajectories of combat units over time. This assumption maintains the number of state variables as the number of asset types by regions throughout simulated time periods. Analysts may use the output files to investigate how the distributions within BEAM change over time. However, BEAM calculates and projects the next day's results based on the current day's data.

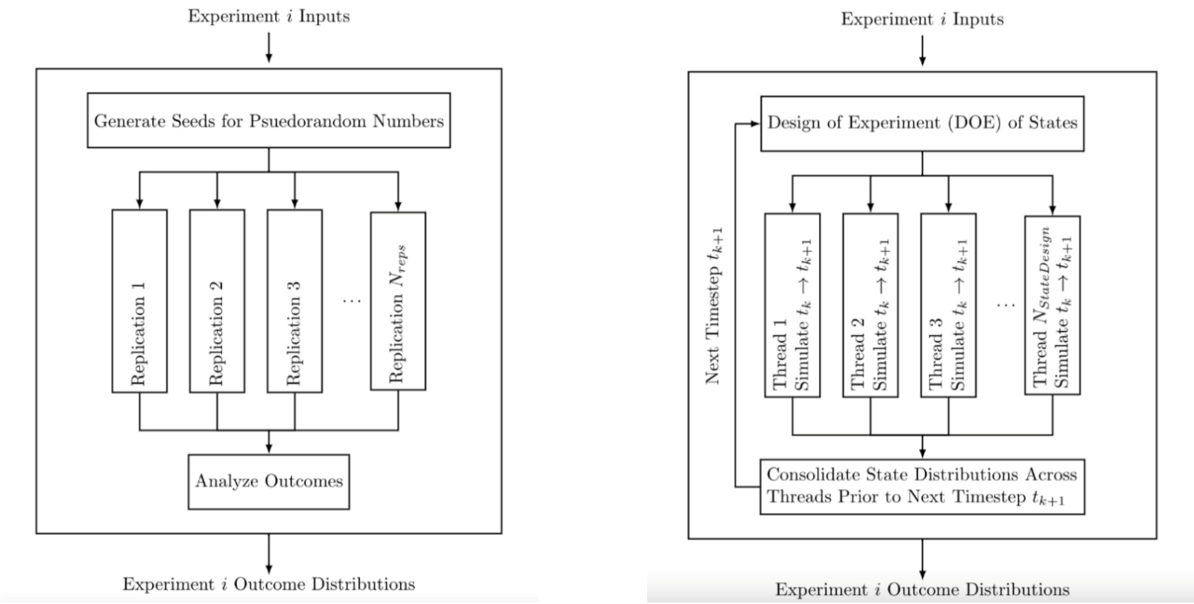
BEAM's breadth includes multi-domain missions from land, maritime, air, and space. With effectiveness data, other missions, such as cyber, may also be incorporated. BEAM's scope includes the functions of offense, defense, intelligence, communications, transportation, and logistic repair.

3.2 BEAM Assessment of Uncertainty

BEAM differs in how it accounts for uncertainty of outcomes. Most complex simulations employ pseudorandom draws based on Monte Carlo random sampling [23], which use replications to assess the uncertainty of outcomes. In contrast, BEAM employs simulation threads that sample the asset-by-region distributions. Each time period, BEAM initiates simulation threads where the initial unit and asset quantities are determined with Quasi-Monte Carlo (QMC) sampling [24]. QMC is categorized as a non-probability or deterministic sampling since the evaluated points are homogeneously equidistantly distributed over the sample space [25]. Our application of QMC selects the points as distributed over the probability space of our asset quantities. Estimation with QMC converges faster than Monte Carlo approaches [26], [27], [28], [25], are unbiased [29], and have become the industry standard in computer graphics [30]. We use the term "simulation threads," which have different input values, to distinguish from clones [31], [32] that start replications with the same initial values.

The traditional approach simulates the entire duration, such as a war in our application. Analysis of the differences between replications determines the uncertainty. In contrast, BEAM uses simulation threads to model the uncertainty. Currently, all the BEAM data is formulated for time-steps of 24 hours representing a simulated day of combat. BEAM inputs include initial asset quantities and military strategies for both sides. The statistical distribution of each asset is tracked within each geographic region. The simulation threads model the system for one cycle or day. After each cycle, the statistical distributions are updated, and a new application of QMC determines asset quantities at which to start new simulation threads for the next simulated day. The new thread quantities in each time period ensure that the computational effort remains focused on the regions of interest; similarly, Etore et al. [33] achieve variance reduction by adapting the strata while sampling.

Figure 3 compares the traditional Monte Carlo approach with replications of the entire combat duration versus the simulation threads where a set of threads models a single time-step.



Replications of entire duration. **Simulation threads of each cycle.**

Figure 3: Comparison of an experiment with replications and with simulation threads.

BEAM generates new threads for each subsequent day. With this novel use of simulation threads, a single pass through simulated time (one run) produces statistical distributions over time for each modeled asset in BEAM. If the combat durations are equal, n replications require the same computation effort as n simulation threads. However, the simulation threads generate better insights because this approach repeatedly spreads the sampling across the reasonable state variable ranges. The QMC approach avoids the Monte Carlo computational burdens of frequently sampling high-probability regions and of occasional outliers distorting the estimates. Since BEAM evaluates a theater campaign in about 15 minutes on a laptop, many experiments of different strategies, forces, and infrastructure may be evaluated.

3.3 BEAM Algorithms for Each Simulated Day

Within each simulation thread for a simulated day, BEAM employs a series of algorithms to model combat. The BEAM software is modular, which enables investigation and improvement. In the following, we describe the current BEAM algorithms in each simulation thread to model combat for a simulated day. BEAM uses the same algorithms for both the Red and Blue sides as shown in Figure 4.

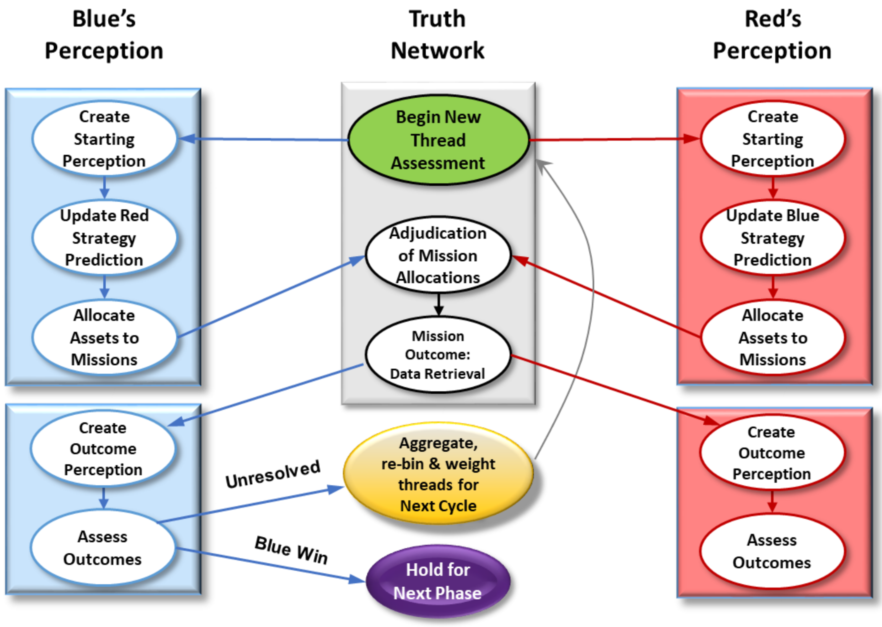


Figure 4: Thread flows within time-step cycles (simulated days) with Blue as the assessing side.

BEAM tracks evolution through time-steps. We evaluate the uncertainty of outcomes by employing a new set of simulation threads for each simulated day. Consistent with the formulation guidelines in [34], we use *lower_case* for fixed input data, *UPPER_CASE* for decision variables, and *Camel_Case* for intermediate variables.

Subscript	Meaning
$a \in A$	Asset types – units, consumables, and infrastructure
$r \in R$	Region – large areas with different combat ends (objectives)
$d \in D$	Simulated day (24-hour period of combat) of a war with duration D
$b \in B$	Equal-probability bin b of B bins for an asset statistical distribution

Superscript	Meaning
$t \in T$	Simulation thread
P	Perception of either Blue or Red
C	Combat phase for a particular side
$CDF^{-1}(Asset_{a,r,d}, X)$	The inverse cumulative distribution function of asset a in region r for simulated day d, evaluated at probability of X.
$Asset_{a,r,d,b}$	The median of each bin for one opponent's assets is a case.
$Asset_{a,r,d}^{t}$	The combination of cases across opponents are the design points at which a new simulation thread t is started.

3.4 BEAM Asset Quantity Distributions

Recall that BEAM's basic modeling entities are missions, which are a collection of offensive assets, adversary defensive assets, and target assets. Losses, either destroyed or consumed, occur to all the assets involved in executed missions; the mission outcomes with the stored probabilities of various losses account for combat uncertainty and result in the updated asset probability distributions. We use distributions with hundreds of step changes to approximate widely varying outcomes. BEAM tracks the probability distribution of each asset type by geographic region.

Analysts know that expected value models are very fast; however, those models do not account for any uncertainty. Monte Carlo simulations address the uncertainty through the computation of replications of the war. The approach in BEAM is between expected value models and replications. BEAM repeatedly employs QMC to generate a small design of experiments. The asset distributions are divided into *b* bins. At each time-step (simulated day), BEAM selects representative design points across the asset distributions corresponding to the *b* bins.

$$Asset_{a,r,d,b} = CDF^{-1}\left(Asset_{a,r,d}, \frac{b-0.5}{B}\right) \tag{1}$$

Initially, the asset quantities are the single input value for the scenario, and the first day, *d*=0, has only one thread. For subsequent days, BEAM employs b^2 cases with Blue ranging from its lowest to highest *b* bin centroids matched with Red similarly ranging across its *b* bin centroids. BEAM starts a new simulation thread at all combinations of Red and Blue bin centroids, $Asset_{a,r,d}^{t}$.

While BEAM is similar to expected value models in that the thread computations are fast, there are two major differences between the two. First, BEAM models each combat adjudication with distributions of various probabilistic outcomes of asset losses. Second, the various threads (with different asset quantities based on the design points) account for the uncertainty of the asset quantities. BEAM threads only last one time-cycle. Each simulated day, BEAM combines the thousands of mission outcomes within each thread and consolidates the asset distributions by region across threads. For the next day, new bins of the asset distributions are calculated with corresponding new threads created and started.

3.5 BEAM Threads

This sub-section describes the BEAM algorithm steps accomplished in each thread in a simulated day [19]. The cycle begins with a truth model that has a statistical distribution for each type of asset in each region. The first day starts with the user-input asset quantities. BEAM applies its internal DOE based on QMC to determine the asset quantities for each thread. For each side, BEAM determines the bin quantity corresponding to the midpoint of the bth quantile for every asset-by-region. The minimum is four bins with 25% probability. Each unit quantity is consistent with the asset distribution for that asset in that region. The thread assets quantities are the combinations of quantile bin quantities across the two opponents. The default of four bins per side results in 16 threads in each cycle (simulated day); the thread asset quantities vary from (Blue-Low, Red-Low) through (Blue-High, Red-High). The user may increase the number of bins, which increases the accuracy through more refined thread quantities at the expense of increased model runtime. A recent test indicates that the outcomes stabilize with 6 bins on each side for a total of 36 simulation threads. Each thread begins with its truth model that has a fixed (usually fractional) number of units by type, region, and side.

An argument against our construction of cases is that, with all the asset quantities for each side increasing between cases, the impactful factor, asset in our case, is statistically confounded. We implemented this approach because the threads are not equally likely because of covariances across the asset quantities. Our combat application exhibits counter-correlation, where generally one side becomes dominant, focusing their assets against the losing side's dwindling assets. Hence, Blue-High with Red-Low (or similarly Blue-Low with Red-High) are much more likely and realistic outcomes than both sides being high or both sides being low. After completing a BEAM period and determining the new bin boundaries, BEAM calculates their associated thread probabilities by categorizing the outcome distributions from the most recently completed period. Therefore, while conceding a slight loss of information due to confounding, we gain significant insight into the appropriate thread probabilities. Furthermore, since losses occur at different rates for various units and systems, BEAM produces results with disparate quantities across its simulated days.

3.6 BEAM Perception Algorithm

For each thread, BEAM processes the simulated day through a series of algorithmic steps, as shown in Figure 4. The Blue and Red sides independently conduct the next steps of perception through allocating asset-to-missions. First, each side has their perception of the adversary strategy (as input by the analyst and then updated through intelligence observations). Each side obtains a perception of the enemy forces based on their ISR capabilities and missions conducted. The perceived quantities are generally a multiple of the true quantity for that thread.

$$Assets^{P}_{a,r,d} = \boldsymbol{Intel}(A_{a,r,d}) assets^{T}_{a,r,d} \tag{2}$$

$\boldsymbol{Intel}(A_{a,r,d})$	The intelligence vector for an asset, by region, on a specific day, maintains the asset portion that are unknown, identified, targetable, struck, and includes the battle damage assessment.
$ends^{C,w}_{a,r,l}$	Ends specify the military objectives in combat Phase C with relative weight w among the various ends. Assets Ends specify the asset-by-region (or group of regions) desired to be driven down to threshold level l. Mission Ends are a collection of Asset Ends where BEAM selects the most efficient Asset Ends to disable that function.
$mission_{a,r}$	Mission m is a collection of offensive assets encountering defensive assets with the goal of impacted targeted assets.
$asset_{a,r,d,t}$	The combination of cases across opponents are the design points at which a new simulation thread t is started. The asset quantities by regions are specified values. BEAM employs b^2 cases with Blue ranging from its lowest to highest bin quantities matched with Red similarly ranging across its bin quantities.

BEAM models missions as packages. Every combination of packages comprises a unique mission m, which is composed of quantities of specific types of assets a. Missions have effects on all the assets involved. The packages are categorized into offensive, defensive, and targeted assets. For offensive missions, the offensive assets are friendly, whereas the defensive and targeted assets belong to the enemy. The mission effects are probabilistic state outcomes (losses) for each asset involved in that mission.

$$\boldsymbol{effects_{ma}} = \begin{cases} \Delta\%_1 & P(state\ 1) \\ \vdots & \vdots \\ \Delta\%_n & P(state\ n) \end{cases} \quad \forall\ a \in m \tag{3}$$

For example, a fighter engaging an air-to-ground mission may have 50% chance of no damage (0, 0.5), 30% probability of 10% losses (-0.1, 0.3), and 20% chance of 20% losses (-0.2, 0.2). Negative effects constitute destruction, attrition, losses, or consumption whereas positive effects represent movements, resupplies, or repairs. These mission effects, usually derived from higher-resolution models, are inputs into BEAM. The allocation algorithm of assets-to-missions optimizes using expected effects; -0.07 fighters lost per mission in the example. The combat adjudication algorithm updates the asset-by-region statistical distributions based on the probabilities of the effect states along with the number of times that mission occurs. For a simulated day, the effects to each asset type in a region across all the missions are summed to update that asset-by-region statistical distribution.

3.7 BEAM Assets-to-Missions Selection Algorithm

The next algorithm selects missions based on the available assets, their strategy, and their perception of their opponent's strategy. The assets-to-missions allocation for each side is a goal-program to best achieve their military strategy, which objective is composed of various ends for each combat phase. An end is a goal specifying a threshold value, which is the desired amount of that type of enemy assets to be operating; the optimization does not credit destruction below this threshold. The decision variables are how many of which missions to execute. The goal-program's objective function weights the accomplishment of their various phase ends based on their assigned importance. For example, with two goals of destroy enemy ground combat at priority 2 and control major supply routes at priority 4, the optimization weights the achievement of controlling major supply routes to its threshold value as twice as important compared to destroying ground combat units. The constraints include limitations of the associated ways, means, and risks. Additional constraints limit the selected missions to less than the available asset quantities.

The adjudication algorithm maximizes the achievement of the priority weighted ends for a specific simulated day and a specified opponent. Let the variable $opponent$ be +1 for friendly and -1 for enemy

$$Max \left[\sum_{friendly\ ends\ e} weight_e\ (opponent) ScoredEffects_{ear} \right] \tag{4}$$

where the quantities of m mission selected, $MSNQTY_m$, determine the expected $ScoredEffects_{ear}$, which limits achievement to the amount specified by the end's threshold.

$$(opponent)\ ScoredEffects_{ear} \leq (opponent) \sum_{m \in M} \mathrm{E}[effects_{ma}]\ MSNQTY_m\ \forall\ a, r\ \in End\ e \tag{5}$$

$$(opponent) ScoredEffects_{ear} \leq (opponent)\ [\ Asset_{ar} - endthreshold_{ear}]\ \forall\ a, r\ \in End\ e \tag{6}$$

The ways specify groups of assets to either include or exclude in executing missions to achieve that end. Ways are implemented by reducing the viable missions. Similarly, means specify a particular asset to use to achieve the ends, further restricting the list of potential missions. Risks are implemented by adding a constraint such that the impacts to the specified asset a are not exceeded.

$$-\sum_{A_{k,j}} \mathrm{E}[effects_{ma}]MSNQTY_m \leq risk_a \quad (7)$$

The allocated missions may not exceed use of available assets in any region.

$$\sum_{m \in M} mission_{a,r} MSNQTY_m \ \leq Asset_{a,r} \ \forall\ a \in A, r \in R \quad (8)$$

Mission effectiveness (in terms of contributing to achieving the ends) depends on what missions, particularly the defensive ones, their adversary employs. Hence, BEAM employs a few rounds of fictitious play. Within each side's perspective of military strategy and available assets, each side alternates Blue and Red allocating assets to missions. This determines their mission $MSNQTY_m$ against the latest round of the opponent's missions. The completion of the fictitious play is that side's asset-to-missions allocation. Each side is independently planning their military strategy and force quantities based on their perception of their adversary. Poor perceptions will likely lead to less desirable performance in the next step of combat adjudication. We contend that BEAM addresses commanders' caution about their uncertainty of the adversary's strength through its other threads that have increased enemy quantities. As in actual combat, both sides continually adjust their actions to best achieve their goals. The adaptation within BEAM avoids the overly optimistic outcomes inherent in most combat simulations that use static or scripted actions for their opponent.

3.8 BEAM Combat Adjudication Algorithm

Within the cycle in the thread, the two sides pass their selected missions to the adjudication algorithm in the truth model. The adjudication algorithm matches up the planned offensive and defensive missions from both sides to determine the actual (simulated) missions that occurred. Whereas the allocation used the expected impacts, the adjudication algorithm employs the probabilistic outcomes. For each mission that occurs, the BEAM mission database provides the probabilities of various losses to the attacking, defending, and targeted assets. Each asset-by-region distribution is updated. Since each mission results in probabilities of a discrete number of consumptions or losses, the resulting cumulative distribution steps are similar to empirical distributions. The various missions increase the number of steps for the employed asset-by-region distributions. BEAM combines all of the thousands of outcomes that affect each side's assets-by-region to generate the updated assets-by-region probability distributions for each thread.

3.9 BEAM Thread Probability Determination

BEAM determines the new threads and their associated probabilities through seven steps. First, within each thread, the assessing side evaluates the probability of having accomplished all of their primary phase objectives. That phase-complete probability, along with the associate asset quantities, are stored and included in the starting asset-by-region distributions for the next phase. Second, the unresolved probabilities are combined across all the threads. Third, for each asset-by-region, BEAM calculates the new bin boundaries for the b equal-probability bins based on the combined unresolved distributions. Fourth, BEAM determines the marginal probability by case from low to high by applying the new overall bin boundaries to the previous thread results for each asset-by-region. Fifth, BEAM sums across these thread probabilities for the threads within each case. Sixth, the overall case is the average of the marginal per case asset-by-region thread probabilities. Seventh, the new thread probabilities are the product of the appropriate Blue and Red case probabilities. Ref. [19] presents a trivial numerical example.

We implemented this process of determining thread probability weights because the unrealistic (unlikely) threads of both sides having high or both sides having low asset quantities does not represent actual combat. Historically, in combat, the dominating side gains a synergistic advantage as their more effective offensive assets are applied against a continually reduced number of defending assets, which produces counter-correlation across the cases between the sides. If the asset case levels were varied within a thread, the averaging in the sixth step would need to correspond to the asset-by-region cases in the new thread. More importantly, the potential dominating effects of the winning side would not be represented across the threads. Hence, while allowing for sampling of the correlation of the contributions of different asset quantities, an approach that varyies asset cases within a thread would eliminate the representation of the synergistic effect of the winning side becoming more and more dominant in combat.

3.10 BEAM Combat Outcome Assessments

Each side gets their perception of the adjudicated outcomes, which is the analytic equivalent of conducting Battle Damage Assessment (BDA). Remember that BEAM is making a single pass through simulated time that accounts for all of the various trajectories of combat. At the end of each simulated day, BEAM has a probabilistic range of outcomes. For the portion of outcomes in which the assessor has achieved all of their primary (assessed) phase ends, BEAM terminates those results and stores those outcomes with their associated probabilities until the next combat phase.

At the conclusion of each simulated day, BEAM's truth model consolidates the unresolved asset-by-region distributions across the threads. These thread outcomes are weighted by the probability of threads, which are not equal; for example, after combat, one side being high and the other low is more likely than both sides being high. We are employing the Markov principle that the history of how combat arrived at those state vectors of asset quantities does not affect future results; hence, this consolidation across threads keeps the state vector of distributions of asset quantities by regions from expanding. The updated distributions become the truth model for the next simulated day.

3.11 BEAM Set Up and Repeats

BEAM repeats the process, starting with creating all new simulation threads for the next simulated day. This cycle continues until either all threads have met the assessors' criteria for the ends defined in that combat phase or the specified duration of the phase occurs. The next combat phase begins, incorporating only the asset quantities and associated probabilities for which all the primary ends in the prior combat phase were achieved.

4.0 BEAM APPLICATIONS

Just like other models in our defense resolution hierarchy [11], which range from engineering to engagement to mission through campaign to enterprise, BEAM requires inputs from the higher-resolution models. In turn, analysts may be able to use BEAM to determine the conditions for campaign simulations or more detailed models. Analysts should use BEAM in conjunction with theater campaign models, as filter to assist in determining the regions of good strategy and force structure to focus the campaign analysis. Similarly, analysts should use mission and campaign models to determine the mission effectiveness inputs in BEAM. Even though BEAM inputs and outputs appear similar to campaign models, BEAM's resolution should limit its use in supporting programmatic decisions. System effectiveness (through mission performance data) is an input to BEAM and not a conclusion to draw from BEAM results. However, demand and importance of units is a reasonable conclusion from BEAM runs.

The importance and challenge of validating military simulations is well known [35]. Poropudas and Virtanen [36] propose validating air combat simulations by comparing simulation, game theory, and actual outcomes. A detailed approach like that is much more difficult at the campaign level. As analysts move up the combat hierarchy of models, they must aggregate inputs often from different models to represent the increased breadth of the more aggregate model [11], [37]. For example, many mission models assume support from space systems, so developing effectiveness inputs for BEAM requires combining mission-level results with other results that reflect the probability of having appropriate space support. BEAM's main modeling entities are missions; hence, high quality mission effectiveness data is a critical step in validation. In a sense, BEAM is just combining the various missions in different ways. We constructed the initial BEAM input data based on the Joint Wargaming Analysis Model (JWAM) [38], [39]. With this data, the BEAM results have face validity in that events occur as expected in the completed study with very diverse settings.

A comparison between BEAM and campaign model results is challenging. We have started developing classified performance data for BEAM, for which we need to have consistent inputs. Another challenge is that within BEAM, both sides adapt as the scenario unfolds. Therefore, a scripted campaign model is unlikely to follow the same trajectory as BEAM. A possibility for comparable scenarios might be to use the BEAM results to script the campaign model flow. Clearly, we have work ahead in validating BEAM results.

A few examples and demonstrations of BEAM's capabilities are described below.

4.1 Force Structure and Effectiveness Studies

BEAM has been used by various organizations to assess force structure. Students at the Air Force Institute of Technology (AFIT) have applied BEAM in two different studies. One class employed BEAM to examine the force mix study [40], and another class applied BEAM to identify impacts of cost-imposing actions [41]. The Naval Postgraduate School (NPS) applied BEAM to conduct a wide-ranging effectiveness study for the US Joint Staff / J7. The US Air Force Futures is starting two studies employing BEAM. The first is to examine the near-term design of the Air Force, and the second study to evaluate the distant future of the US Air Force and Space Force. The US Center for Army Analysis (CAA) is also applying BEAM to scope a major wargame.

4.2 Strategy Exploration Studies

In addition to these force structure studies, we demonstrated BEAM's ability to explore a wide range of military strategies. Prior to BEAM, the analytic community did not have the means to search a wide range of operational theater strategies. In the following notional scenario, we first evaluated the probability that Red could achieve their objectives of occupying an adjacent nation, "Desired Red Land (DRL)." Subsequently, a more sophisticated strategy analysis is conducted to include probability of success along with two additional metrics: casualties and duration of combat. In both these examples, the forces are fixed and only the impacts of changing military strategies are considered.

4.2.1 Time-Constrained Strategy

The first exploration focused on a time-constrained strategy where phase lengths were restricted. These durations often impacted Red's ability to move on to the next phase. The basic strategy, along with maximum phase durations for this study, are as follows:

- Phase 1: Attack Integrated Air Defense Systems (IADS) in DRL – 2 Days
- Phase 2: Attack Tactical Air (TAC Air) Assets and IADS in DRL – 3 Days
- Phase 3: Bombard blue troops / air assets in DRL – 5 Days
- Phase 4: Take Land and Mission Supply Routes (MSR) in DRL – 12 Days

The IADS units consist of radar stations, since surface-to-air missiles and launchers are modeled separately. The Tactical Air units include both fighter and attack helicopter squadrons. In this case, Red's achievement percentage of all their ends is used to assess the best strategy. Strategy exploration focused on adjusting the thresholds (or level of degradation required to move to the next phase) for the IADS and Tactical Air Asset Ends. Requiring too much degradation lowers the success rate in early phases (which greatly impacts overall success rate, since threads that do not meet the thresholds are terminated) and might degrade Red assets (particularly missiles) too early in the campaign. Conversely, if IADS and Tactical Air assets are not degraded enough, success rate in later phases may decline.

Table 1 shows the results of this analysis with success rate displayed at the overall and individual phase levels. The baseline strategy requires the highest level of degradation of an IADS radar unit and Tactical Air, which is a fighter squadron, degrading the IADS unit to 0.5 and Tactical Air to 1.5 squadrons by the end of Phase 2. This results in a 34% overall success rate. Recall that only the successful threads progress forward to the next phase, so the success rates are multiplicative. That is, the overall success rate is the product of each individual phase success rate (76% x 64% x 100% x 70% = 34%).

We evaluate three courses of action (COAs) in this study by varying the thresholds for the ends in phases 1 and 2. COA 1, which relaxed the Phase 1 and 2 IADS thresholds by 0.5, led to the highest overall completion percentage. COA 2 (which relaxed the TAC Air threshold in Phase 2) and COA 3 (which relaxed all the thresholds) both led to higher completion rates than the baseline but did not outperform COA 1. With COA 1, Red improved from 34% to 76% success with no changes to their force's structure. BEAM's ability to analyze ends' thresholds quickly and easily can lead to significant improvement in overall success rate.

Table 1: Red strategy exploration (time constricted scenario).

	Red Achievement Percent							
	Overall	Phase 1	Phase 2	Phase 3	Phase 4	Phase 1 IADS Threshold	Phase2 IADS Threshold	Phase 2 Tac Air Threshold
Baseline	34%	76%	64%	100%	70%	1	0.5	1.5
COA 1	76%	92%	86%	100%	96%	1.5	1	1.5
COA 2	61%	76%	100%	100%	80%	1	0.5	2.5
COA 3	53%	92%	100%	100%	57%	1.5	1	2.5

In the military strategies evaluated above, Blue was in a purely defensive posture, allocating all of its assets to protecting DRL without attacking Red Land. We next evaluate various offensive strategies Blue could employ against Red COA 1. Similar to Red's offensive strategy, we consider the impact of Blue attacking Red's IADS and Tactical Air assets.

The results of Blue offensive attacks, shown in Table 2, have a major impact on Red's ability to achieve its campaign objectives. An analysis of the results shows that Blue was not actually successful in significantly degrading Red assets. Rather, Red was forced to adapt and allocate assets to defensive missions, which it did not have to do before. The re-allocation of missions decreased Red's ability to execute offensive missions and their probability of success decreased from 76% to 6%. Hence, with only changes in Blue's military strategy, Red's overall success was reduced significantly!

Table 2: Blue offensive strategy exploration.

	Blue Strategy (vs. Red COA 1)	Red Achievement Percent				
		Overall	Phase 1	Phase 2	Phase 3	Phase 4
Red COA 1	Defensive Posture	76%	92%	86%	100%	96%
COA 1	Attack IADS	6%	44%	16%	88%	98%
COA 2	Attack Tac Air	6%	44%	15%	95%	99%
COA 3	Attack IADS and Tac Air	6%	44%	15%	95%	100%

4.2.2 Event Based Strategy

A major constraint in the strategy exploration presented in the previous section was the short phase durations. Another way to evaluate strategy is to extend phase lengths so their durations have no impact on the completion percentage. Next, we show results of a study that used the same scenario but extended the phases to reduce their impact on completion percentage. In this case, we investigate the impacts that different phase structures and risk acceptance levels can have on success. The COAs evaluated for this study are as follows:

- COA 1 (single phase): Red immediately sends troops into DRL.
- COA 2 (three phases):
  - Phase 1: Red attacks Blue IADS in DRL using Air and Missile Strikes.
  - Phase 2: Red bombards DRL ground troops and TAC Air using Air and Missile Strikes.
  - Phase 3: Red sends in ground troops to seize Blue Land and MSRs in DRL.
- COA 3 (COA 2 with reduced risk): Red preserves air assets by accepting less risk.

To begin, we evaluate Red's chances of success if troops are sent in to seize DRL with no initial bombardment activity. The goal in this COA was for Red to seize Land Area and MSRs. As you can see in Figure 5, after 60 days of combat Red has not made much progress; although they have had some success in seizing MSRs from Blue and may be able to achieve that end given more time, very little Blue Land Area was seized. In this case, Red was not able to overcome Blue forces and seize DRL without a bombardment phase prior to sending in troops. In this case, casualties and combat duration are not evaluated because Red never achieves their invasion.

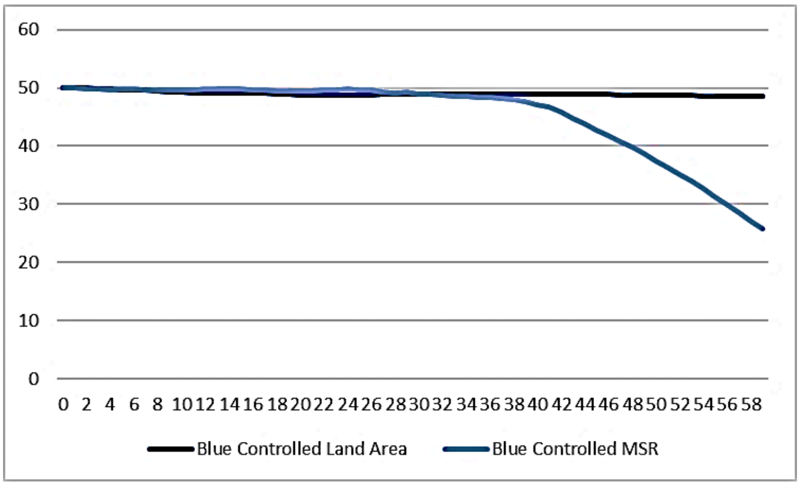


Figure 5: COA 1 immediate Red invasion.

With bombardment and plenty of time, Red eventually achieves their objectives so achievement percent is less critical. The main metrics to evaluate the strategy are completion time and casualties. In COA 2, we added two phases before sending in ground forces. The first phase consists of air and missile strikes on Blue's IADS in DRL. The second phase targets Blue ground troops and Tactical Air assets. Additionally, we evaluate success at various IADS thresholds and selected the best outcome. Adding the two phases before the ground attack led to a 100% success rate for Red, regardless of the IADS threshold. The best results for Red, highlighted in Table 3 had the fewest Red casualties (5,398) and relatively shorter completion time (26.6 days).

Through additional analysis of results, we discovered that Red lost most of its Tactical Air assets and bombers in the first two phases of combat. Figure 6 illustrates those losses. As a result, we examine a third COA that limits the risk to these assets in the first two phases.

Table 3: IADS threshold Design of Experiment (DOE).

IADS Threshold	Achievement	Completion Time	Red Casualties	Blue Casualties
0.75	100%	28.9	5914	8135
1	100%	27.1	5592	8362
1.25	100%	26.9	5743	8326
1.5	100%	26.6	5398	8333
1.75	100%	26.1	5604	8284
2	100%	26.5	5511	8526

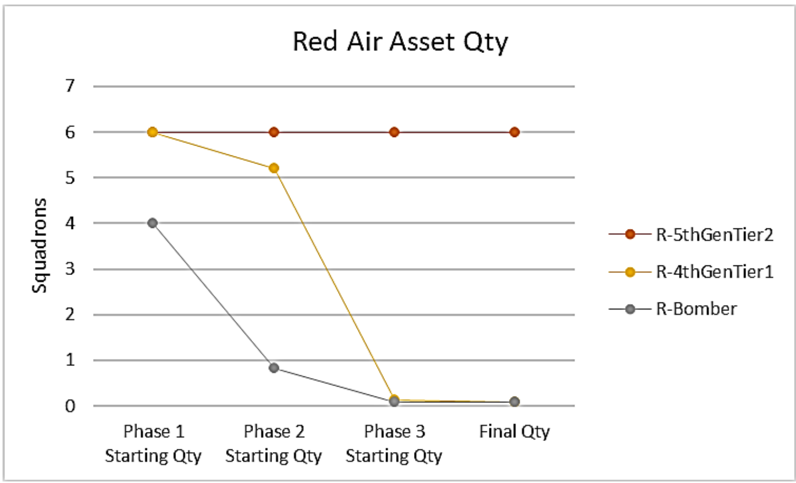


Figure 6: Red Air Asset Quantities by Phase (COA 2).

In COA 3, we add a risk strategy (constraint) that protects Bombers and 4th Generation Fighters in the first two phases, so BEAM limited these assets to standoff missions. Table 4 shows that this COA actually led to a faster completion (25.7 days) with fewer Red casualties (5,293) than COA 2. Additionally, Figure 7 shows that, not only were Bombers and 4th Generation Fighters available to support ground troops in Phase 3, but Red also had 1.5 4th Generation Fighter squadrons remaining at the end of the conflict (compared to zero remaining in COA 2).

Table 4: Red's COA results with risk limits for fighters and bombers.

	IADS Threshold	Achievement	Completion Time	Red Casualties	Blue Casualties
COA 1	N/A	0%	N/A	N/A	N/A
COA 2	1.5	100%	26.6	5398	8333
COA 3	0.75	100%	26.2	5676	8331
	1	100%	27	5620	8146
	1.25	100%	27.6	5805	8247
	1.5	100%	30.9	6625	8089
	1.75	100%	25.7	5293	8477
	2	100%	25.5	5372	8491

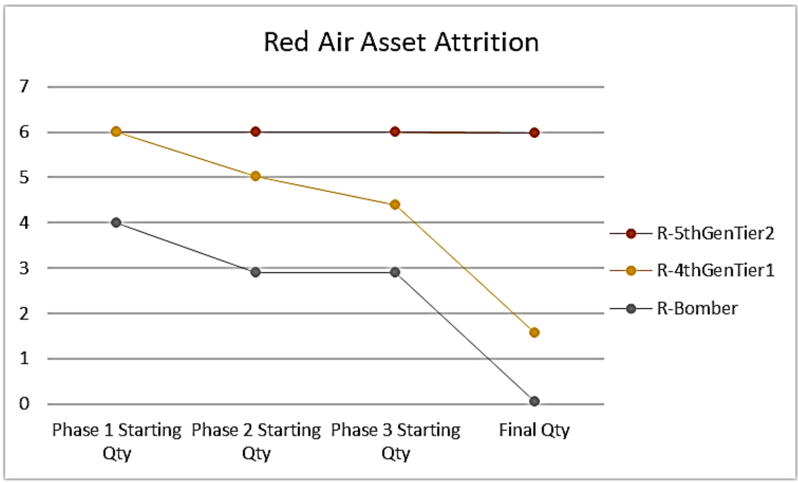


Figure 7: Red air asset quantities by phase (COA 3).

Similar to the previous study, Blue is in a purely defensive posture, so we consider two Blue COAs against Red COA 3.

- Blue COA 1: Blue attacks Red runways in Red Land.
- Blue COA 2: Blue increases defensive air missions in DRL.

Although we recommend a more thorough strategy search, the results seen in Table 5 suggest that Blue should consider launching offensive attacks on Red runways. These attacks reduce Red's achievement percent (95%), increase completion time (29.2 days), and lead to more Red casualties (5,345) and fewer Blue casualties (8,034) than seen in Red COA 3 results.

This hypothetical scenario with fixed forces demonstrates the critical importance of military strategy. BEAM enables military planners to evaluate and refine their theater ends, ways, means, and risks.

Table 5. Blue COA exploration against Red COA 3.

	IADS Threshold	Achievement	Completion Time	Red Casualties	Blue Casualties
Red COA 3	1.75	100%	25.7	5293	8477
Blue COA 1	1.75	95%	29.17	5345	8034
Blue COA 2	1.75	100%	26.94	6052	8299

5.0 CONCLUSIONS

BEAM was developed in response to the defense analytic community's need to respond to our leaders, particularly to their questions about military strategy. Compared to campaign simulations, which model individual units and platforms, BEAM's primary modeling entities are missions. BEAM limits mission selection to what that side's available assets can accomplish. BEAM employs a novel approach to simulation. Researchers [26], [27], [28], [25] demonstrated that QMC sampling converges faster to unbiased solutions than Monte Carlo techniques. Rather than the traditional Monte Carlo approach that completes multiple replications from which analysts calculate aggregated statistics, our QMC approach applies a set of non-probabilistic (deterministic) sampled threads with non-equal probabilities and daily re-aggregation of probability distributions to represent the uncertainty in combat. BEAM models time-steps or cycles that represent a simulated day with multiple simulation threads, producing the distribution of outcomes with one pass through simulated time.

BEAM is best suited to address issues that include:

- Exploring large trade-spaces of possible joint forces and strategies.
- Investigating diverse scenarios including concurrent conflicts.
- Assessing how opponents may adapt in response to an adversary's enhancements.
- Determining mission areas where improvements significantly alter campaign success.
- Exploring mission capability improvements (% increases in damage-inflicted or survivability) that can alter scenario outcomes.
- Allowing user interaction inside scenarios to support campaign wargames.

BEAM is not intended to conduct system performance trades or detailed mission or engagement questions. Mission effects as determined by system performance are inputs, so BEAM cannot provide insights on system characteristics. Similarly, any questions that involve knowledge of specific system locations are below BEAM's resolution. BEAM estimates the likelihood of achieving campaign objectives, as specified by ends for each combat phase, against an adaptive adversary. BEAM does assess the demand of units or systems and their projects losses. Analysts gain insights into how changes in unit and system quantities, infrastructure, and strategy (to include perceptions of adversary strategies) can impact campaigns. Analysis with BEAM should support a process that leads to more detailed studies of promising ideas.

The advantages of BEAM are that it is easy to use, computationally fast, and flexible. Analysts new to BEAM can begin modifying existing scenarios within hours. New BEAM users can build scenarios with a couple of weeks of experience. BEAM runs on a laptop with a scenario evaluation generally requiring 5 to 30 minutes depending on the size of the scenario. BEAM is flexible in that constructing a new scenario simply requires estimates of the order of battle (force quantities) and projections of the strategies for both sides.

The design goal is an open architecture, so members of the community may insert and test different algorithms within BEAM's modular structure. The calendar year 2023 enhancements focused on data management so users can better manipulate inputs, particularly adding new weapon systems. Planned BEAM development includes producing classified mission effects data, including logistics and cyber missions. We also intend to use BEAM in campaign wargames exploring operational-level decisions.

BEAM is a government model that is available to the defense analytic community. The US Department of Defense funds BEAM's development and maintenance, and the US government has unlimited rights to the software and data. They distribute BEAM to the defense analytic community. The BEAM software comes with the mission database, user manual, technical documentation, and training videos. The Air Force recently certified BEAM for use on government networks. The BEAM software has currently been distributed for evaluation to every service, foreign partners, Federally Funded Research and Development Centers (FFRDCs), contractors, and academia. The Foreign Disclosure Office has cleared BEAM for distribution to our NATO partners and Five Eyes countries. NATO is considering BEAM to be part of the NATO Next Gen M&S. The Naval Postgraduate School in support of a Joint Staff / J7 study used BEAM in its first operational application. Currently, Air Force Futures is conducting two studies with BEAM, and the Center for Army Analysis (CAA) is using BEAM to develop a war game scenario. You may request BEAM through BEAM@Linquest.com.

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